Temporal Logic

Oct 17, 2019

Introduction

- Classical Logic:
  - Good for describing static conditions.

Wumpus world

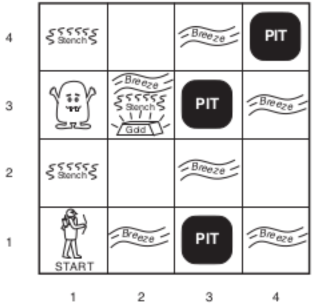


Figure 7.2 A typical wumpus world. The agent is in the bottom left corner, facing right.

Wumpus world

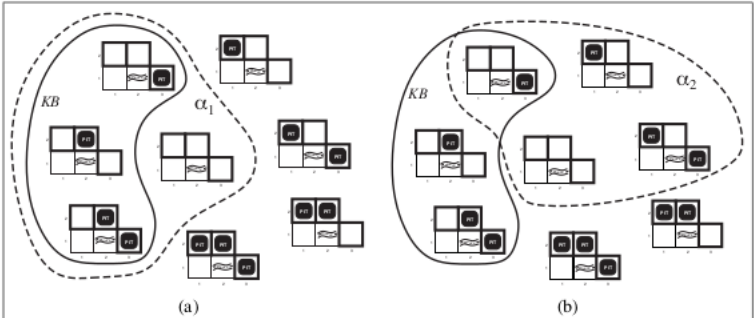
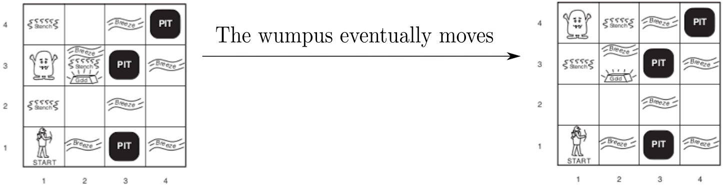


Figure 7.5 Possible models for the presence of pits in squares [1,2], [2,2], and [3,1]. The KB corresponding to the observations of nothing in [1,1] and a breeze in [2,1] is shown by the solid line. (a) Dotted line shows models of α_1 (no pit in [1,2]). (b) Dotted line shows models of α_2 (no pit in [2,2]).

Wumpus world

Now consider the situation where wumpus can move.



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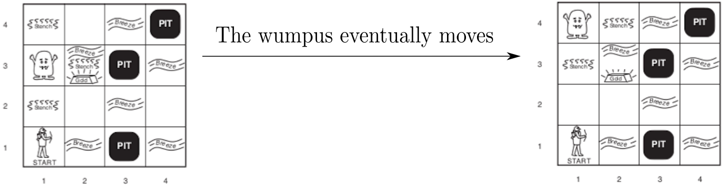

Can we represent this using PL?

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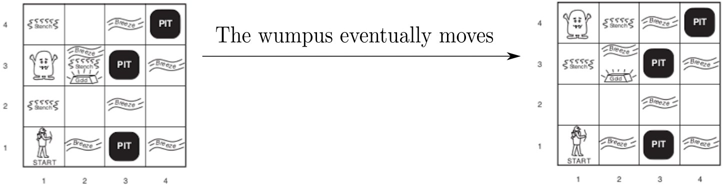

Can we represent this using PL? No
By the end of the class we will see how to represent this case using TL.

What can you do with TL?

- Temporal Logic:
  - Adds temporal operators.
  - Describe how static conditions change over time.

Logics

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Figure 8.1 Formal languages and their ontological and epistemological commitments.

Temporal Logic

Syntactic aspects

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In TL, as well as propositional operators, we use temporal operators referring to moments in the *future*:

Formula	**Intuitive Meaning**
$\bigcirc\varphi$	φ is true in the *next* moment in time.
$\Box\varphi$	φ is true in *all* future moments.
$\Diamond\varphi$	φ is true in *some* future (or present) moment.
$\varphi U \psi$	φ continues being true *up until* some future moment when ψ is true.
$\varphi W \psi$	φ continues being true *unless* ψ becomes true.

Formal definition

Formulae in TL are constructed from the following elements:

- A finite set of propositional symbols, typically represented by lower case alphanumeric strings. such as $p, q, r, \mathit{trigger}, \mathit{lunch}, \cdots$
- Propositional connectives: $\mathit{true}, \mathit{false}, \neg, \vee, \wedge, \Leftrightarrow$, and $\Rightarrow$.
- Temporal connectives: $\mathit{start}, \bigcirc, \Diamond, \Box$, U, and W.
- Parenthesis, '(' and ')', generally used to avoid ambiguity.

Formal definition

Set of well-formed formulae of TL, denoted by WFF, is now inductively defined as the smallest set satisfying the following rules:

- Any set of propositional symbols is in WFF.
- *true*, *false* and *start* are in WFF.
- If φ and ψ in WFF, then so are
$$\neg\varphi \quad \varphi \land \psi \quad \varphi \lor \psi \quad \varphi \Rightarrow \psi \quad \varphi \Leftrightarrow \psi \quad (\varphi)$$
$$\Diamond\varphi \quad \Box\varphi \quad \varphi U \psi \quad \varphi W \psi \quad \bigcirc\varphi.$$

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- "In the second moment in time, '*hot*' will be true.":

$$start \Rightarrow \bigcirc hot$$

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Examples

- "*wet* is equivalent to not *dry*":

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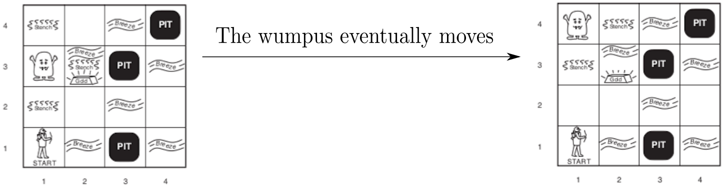
$$\Box wet \Leftrightarrow \neg \Diamond dry$$

Wumpus world



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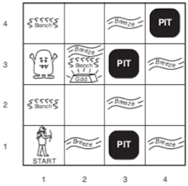

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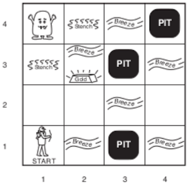

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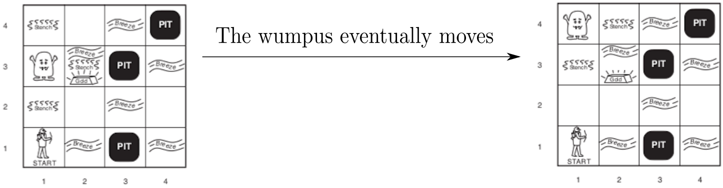

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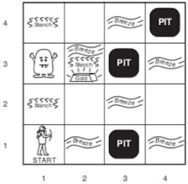

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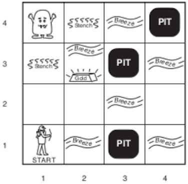

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$$\Box \mathit{wumpus\_moves}$$

Consider the following two expressions:

- $\Diamond(\Box wumpus\_moves)$
- $\Box(\Diamond wumpus\_moves)$

Are both the expressions same or different?