Temporal Logic

Introduction

What can you do with TL?

Logics

Temporal Logic

Syntactic aspects

Formal definition

Examples

Always eventually and eventually always

Temporal Logic

Oct 17, 2019
Introduction

- Classical Logic:
  - Good for describing static conditions.
Wumpus world

Figure 7.2  A typical wumpus world. The agent is in the bottom left corner, facing right.
Wumpus world

Figure 7.5  Possible models for the presence of pits in squares [1,2], [2,2], and [3,1]. The KB corresponding to the observations of nothing in [1,1] and a breeze in [2,1] is shown by the solid line. (a) Dotted line shows models of $\alpha_1$ (no pit in [1,2]). (b) Dotted line shows models of $\alpha_2$ (no pit in [2,2]).
Wumpus world

Now consider the situation where wumpus can move.
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Wumpus world

Now consider the situation where wumpus can move.

Can we represent this using PL?
Wumpus world

Now consider the situation where wumpus can move.

Can we represent this using PL? No
By the end of the class we will see how to represent this case using TL.
What can you do with TL?

- **Temporal Logic:**
  - Adds temporal operators.
  - Describe how static conditions change over time.
Logics

<table>
<thead>
<tr>
<th>Language</th>
<th>Ontological Commitment (What exists in the world)</th>
<th>Epistemological Commitment (What an agent believes about facts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>First-order logic</td>
<td>facts, objects, relations</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Temporal logic</td>
<td>facts, objects, relations, times</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Probability theory</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>facts with degree of truth ∈ [0, 1]</td>
<td>degree of belief ∈ [0, 1]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>known interval value</td>
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**Figure 8.1** Formal languages and their ontological and epistemological commitments.
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Temporal Logic
In TL, as well as propositional operators, we use temporal operators referring to moments in the *future*:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Intuitive Meaning</th>
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<tbody>
<tr>
<td>$\circ \phi$</td>
<td>$\phi$ is true in the next moment in time.</td>
</tr>
<tr>
<td>$\Box \phi$</td>
<td>$\phi$ is true in all future moments.</td>
</tr>
<tr>
<td>$\Diamond \phi$</td>
<td>$\phi$ is true in some future (or present) moment.</td>
</tr>
<tr>
<td>$\phi \mathbin{U} \psi$</td>
<td>$\phi$ continues being true up until some future moment when $\psi$ is true.</td>
</tr>
<tr>
<td>$\phi \mathbin{W} \psi$</td>
<td>$\phi$ continues being true unless $\psi$ becomes true.</td>
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In TL, as well as propositional operators, we use temporal operators referring to moments in the future:

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<td>◊φ</td>
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<tr>
<td>φUψ</td>
<td>φ continues being true up until some future moment when ψ is true.</td>
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<tr>
<td>φWψ</td>
<td>φ continues being true unless ψ becomes true.</td>
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Formal definition

Formulae in TL are constructed from the following elements:

- A finite set of propositional symbols, typically represented by lower case alphanumeric strings, such as $p, q, r, trigger, lunch, \cdots$
- Propositional connectives: true, false, $\neg, \lor, \land, \leftrightarrow, \text{ and } \Rightarrow$.
- Temporal connectives: start, $\bigcirc, \Diamond, \square, U, \text{ and } W$.
- Parenthesis, ‘(’ and ‘)’, generally used to avoid ambiguity.
Set of well-formed formulae of TL, denoted by $WFF$, is now inductively defined as the smallest set satisfying the following rules:

- Any set of propositional symbols is in $WFF$.
- $true$, $false$ and $start$ are in $WFF$.
- If $\varphi$ and $\psi$ in $WFF$, then so are
  
  $\neg \varphi$  $\varphi \wedge \psi$  $\varphi \vee \psi$  $\varphi \Rightarrow \psi$  $\varphi \Leftrightarrow \psi$  $\varphi$  $\varphi$  $\varphi U \psi$  $\varphi W \psi$  $\varphi$. 


Examples

Which of the following are legal $WFF$ of TL:
Examples

Which of the following are legal WFF of TL:

- $pU(q \land \diamond r)$:
Examples

Which of the following are legal *WFF* of TL:

- \( pU(q \land \Diamond r) \): legal
Examples

Which of the following are legal $WFF$ of TL:

- $pU(q \land \diamond r)$: legal
- $p\diamond q$:
Examples

Which of the following are legal *WFF* of TL:

- $p \mathcal{U} (q \land \Diamond r)$: legal
- $p \Diamond q$: not legal
Examples

Which of the following are legal $WFF$ of TL:

- $pU(q \land \diamond r)$: legal
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- $(Ur)$:
Examples

Which of the following are legal WFF of TL:

- $pU(q \land \Diamond r)$: legal
- $p\Diamond q$: not legal
- $(Ur)$: not legal
Examples

Which of the following are legal WFF of TL:

- \( pU(q \land \lozenge r) \): legal
- \( p\lozenge q \): not legal
- \( (Ur) \): not legal
- \( (f \land \Box g)U\lozenge\Box\neg h \):
Examples

Which of the following are legal WFF of TL:

- \( pU(q \land \Diamond r) \): legal
- \( p\Diamond q \): not legal
- \( (Ur) \): not legal
- \( (f \land \Box g)U\Diamond \Box \neg h \): legal
Examples

Which of the following are legal WFF of TL:

- $pU(q \land \diamond r)$: legal
- $p\diamond q$: not legal
- $(Ur)$: not legal
- $(f \land \Box g)U\diamond \Box \neg h$: legal
- $\Box \text{july} \land \text{august}(\diamond \text{september})$: 
Examples

Which of the following are legal WFF of TL:

- $pU(q \land \diamond r)$: legal
- $p\diamond q$: not legal
- $(Ur)$: not legal
- $(f \land \bigcirc g)U\diamond \Box \neg h$: legal
- $\bigcirc july \land august(\diamond september)$: not legal
Examples

Which of the following are legal WFF of TL:

- $pU(q \land \diamond r)$: legal
- $p\diamond q$: not legal
- $(Ur)$: not legal
- $(f \land \lozenge g)U\lozenge \square \neg h$: legal
- $\lozenge july \land \text{august}(\diamond \text{september})$: not legal
- $\text{may} \lor \lozenge ((\text{april}W\text{may}))$: 
Examples

Which of the following are legal WFF of TL:

- $p \mathbf{U} (q \land \lozenge r)$: legal
- $p \lozenge q$: not legal
- $(U r)$: not legal
- $(f \land \circ g) \mathbf{U} \lozenge \Box \neg h$: legal
- $\circ july \land august(\lozenge september)$: not legal
- $may \lor \circ ((april \lor may))$: legal
Examples

Let’s try to represent a real life scenario.
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- “whenever we try to print a document then at some future moment we will not try to print it.”:
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“whenever we try to print a document then at some future moment we will not try to print it.”:

\(\square(\text{try\_to\_print} \Rightarrow \Diamond \neg \text{try\_to\_print})\)
Examples

Let’s try to represent a real life scenario.

- “Whenever we try to print a document then at some future moment we will not try to print it.”:
  \[ \square (\text{try to print} \Rightarrow \Diamond \neg \text{try to print}) \]

- “Whenever we try to print a document then, at the next moment in time either the document will be printed or we try to print again”: 
Examples

Let’s try to represent a real life scenario.

- “whenever we try to print a document then at some future moment we will not try to print it.”:

  \[ \Box (\text{try\_to\_print} \Rightarrow \Diamond \neg \text{try\_to\_print}) \]

- “whenever we try to print a document then, at the next moment in time either the document will be printed or we try to print again”:

  \[ \Box (\text{try\_to\_print} \Rightarrow \bigcirc (\text{printed} \lor \text{try\_to\_print})) \]
Examples

Let’s look at examples using the syntax.

"If a message is sent to a receiver, then the message will eventually be received":

\[ \text{send message} \Rightarrow \lozenge \text{receive message} \]

"It is always the case that, if either 'have passport' or 'have ticket' is false, then, in the next moment of time, 'board flight' will also be false":

\[ \Box ( (\neg \text{have passport} \lor \neg \text{have ticket}) \Rightarrow \Diamond \neg \text{board flight}) \]
Examples

Let’s look at examples using the syntax.

- “If a message is sent to a receiver, then the message will eventually be received”:
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- “If a message is sent to a receiver, then the message will eventually be received”:

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- “It is always the case that, if either ‘have\_passport’ or ‘have\_ticket’ is false, then , in the next moment of time ‘board\_flight’ will also be false”:

  \[ \square \left( \neg \text{have\_passport} \lor \neg \text{have\_ticket} \right) \Rightarrow \lozenge \neg \text{board\_flight} \]
Examples

Let’s look at examples using the syntax.

- “If a message is sent to a receiver, then the message will eventually be received”: 

  \[ send\_message \Rightarrow \diamond receive\_message \]

- “It is always the case that, if either ‘have_passport’ or ‘have_ticket’ is false, then, in the next moment of time ‘board_flight’ will also be false”: 

  \[ \square((\neg have\_passport \lor \neg have\_ticket) \Rightarrow \bigcirc \neg board\_flight) \]
Examples

- “If someone is born, then it is living up until the point in time that it becomes dead”: 

```plaintext
(born ⇒ living) ∧  ♦ dead
```

“In the next moment in time, ‘running’ will be true and, at some time after that, ‘terminated’ will be true.”:

```plaintext
(♦ running) ∧ ♦ terminated
```
Examples

■ “If someone is born, then it is living up until the point in time that it becomes dead”:

\[ \text{born} \Rightarrow \text{livingUdead} \]
Examples

- “If someone is born, then it is living up until the point in time that it becomes dead”:

  $$born \Rightarrow living \land \neg dead$$

- “In the next moment in time, ‘running’ will be true and, at some time after that, ‘terminated’ will be true.”:
Examples

- “If someone is born, then it is living up until the point in time that it becomes dead”:
  
  \[ \text{born} \Rightarrow \text{livingUdead} \]

- “In the next moment in time, ‘running’ will be true and, at some time after that, ‘terminated’ will be true.”:
  
  \[ \bigcirc (\text{running} \land \bigcirc \Diamond \text{terminated}) \]
Examples

- “There is a moment in the future where either pink is always true, or brown is true in the next moment in time”:
Examples

“There is a moment in the future where either pink is always true, or brown is true in the next moment in time”:

\[ \Diamond (\Box \text{pink} \lor \bigcirc \text{brown}) \]
Examples

- “There is a moment in the future where either pink is always true, or brown is true in the next moment in time”:
  \[ \Diamond (\Box pink \lor \Diamond brown) \]

- “In the second moment in time, ‘hot’ will be true.”:
Examples

■ “There is a moment in the future where either pink is always true, or brown is true in the next moment in time”:

\[ \Diamond (\Box \text{pink} \lor \Diamond \text{brown}) \]

■ “In the second moment in time, ‘hot’ will be true.”:

\[ \text{start} \Rightarrow \Diamond \text{hot} \]
“When I start_lecture it implies that I have to talk up until the time to end_lecture”: 
Examples

“When I start_lecture it implies that I have to talk up until the time to end_lecture”:

\[ \text{start}_\text{lecture} \Rightarrow \text{talk}\text{Uend}_\text{lecture} \]
Examples

- “When I start_lecture it implies that I have to talk up until the time to end_lecture”:

  \[\text{start}_\text{lecture} \Rightarrow \text{talk} \& \text{end}_\text{lecture}\]

- “If counter is less_than_7 keep increasing until it is more_than_7”:
Examples

- “When I start_lecture it implies that I have to talk up until the time to end_lecture”:

  \[ \text{start_lecture} \Rightarrow \text{talkUend_lecture} \]

- “If counter is less_than_7 keep increasing until it is more_than_7”:

  \[ \text{less_than_7} \Rightarrow \text{increasingUmore_than_7} \]
Examples

- “wet is equivalent to not dry”:
Examples

“*wet* is equivalent to not *dry*”:

\[ \Box \text{wet} \iff \neg \Diamond \text{dry} \]
### Wumpus world

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PIT</td>
<td>PIT</td>
<td>PIT</td>
</tr>
<tr>
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<td>PIT</td>
<td>PIT</td>
<td>PIT</td>
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The wumpus eventually moves

<table>
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<td>PIT</td>
</tr>
</tbody>
</table>
Wumpus world

The wumpus eventually moves

◊ wumpus_moves
Wumpus world

The wumpus eventually moves

◊ \textit{wumpus moves}

Now what if the wumpus always moves
Wumpus world

The wumpus eventually moves

◊ wumpus\_moves

Now what if the wumpus always moves

□ wumpus\_moves
Consider the following two expressions:

- $\Diamond (\Box \textit{wumpus}\_\textit{moves})$
- $\Box (\Diamond \textit{wumpus}\_\textit{moves})$

Are both the expressions same or different?