Fuzzy Logic

• Today’s class:
  No notes, but the following are useful resources:
  http://www.seattlerobotics.org/Encoder/mar98/fuz/flindex.html
  http://plato.stanford.edu/entries/logic-fuzzy/

• Objectives:
  – Broaden FOL treatment in R&N.
  – Given a description of a set of behaviors and their fuzzy input
    and output sets, combine the behaviors using the centroid of
    area and centroid of largest area methods.
  – Be able to evaluate fuzzy rules with conjunctions,
    disjunctions, and negations
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Fuzzy Logic

• What if the precise value of the vector output wasn’t important?
  – The opposite of “precise” is “fuzzy”

• Turn HARD_RIGHT instead of -90 deg

• Go FAST instead of 0.8*Vmax

• This may be more natural, more linguistic

How do we generate and combine outputs like HARD_RIGHT and SOFT_LEFT?
General Procedure

- **Generate fuzzy outputs (Fuzzification)**
  - Usually have perception operations that yields a “crisp” number that is then converted to a fuzzy input (fuzzified)

- **Apply fuzzy rules (Fuzzy Rules)**
  - The control logic has rules to convert the fuzzy input into a fuzzy output

- **Combine fuzzy outputs (Defuzzification)**
  - When multiple components produce fuzzy outputs, they have to be converted to a single “crisp” number
Motivating Example: Swerving Robot

• “Swerve” is a runaway behavior that doesn’t let the robot turn more than 90 degrees from current direction
  – Vdir: [0,90]
  – Closer the robot, the harder the turn to the right
• Robot velocity is same (for simplicity)
Linguistic-based rules

• Swerve 1 (from sensor 1)
  – If an obstacle is close, take a HARD RIGHT

• Swerve 2 (from sensor 2)
  – If an obstacle is far, take a SOFT RIGHT
Fuzzy Sets: Domain

CLOSE is a fuzzy set over the domain

Crisp domain: PROXIMITY
Distance from robot

Degree of membership

CLOSE
Fuzzy Sets: Multiple Sets

Notes:

- Fuzzy sets often overlap -- that’s seen as a Good Thing
- Set can have different shapes (lines, trapezoids, sigmoids)
Membership Functions

Crisp domain: PROXIMITY
Distance from robot

If robot is 3 meters from obstacle, it has a membership in CLOSE of 0.7 and a membership in FAR of 0.3

\[ M_{\text{CLOSE}}(3) = 0.7 \quad M_{\text{FAR}}(3) = 0.3 \]
Back to Swerve

- Perceptual schema is a tuple
  - membership function for the fuzzy variable PROXIMITY
    - $M_{CLOSE}(x)$, $M_{FAR}(X)$
Motor Schema might be expressed as rule(s):

- **Fuzzy rule**
  - If PROXIMITY is CLOSE
    - TURN is HARD_RIGHT

- **Fuzzy rule**
  - If PROXIMITY is FAR
    - TURN is SOFT_RIGHT
Fuzzy Output Variable

Crisp domain: TURN
Distance for robot to turn to the right

Degree of membership

SOFT_RIGHT
HARD_RIGHT

0 1
0 90
Motor Schema might be expressed as rule(s):

- If PROXIMITY is CLOSE
  - TURN is HARD_RIGHT
  \[ M_{\text{CLOSE}}(3) = 0.7 \]

- If PROXIMITY is FAR
  - TURN is SOFT_RIGHT
  \[ M_{\text{FAR}}(3) = 0.3 \]
Resulting Membership

Crisp domain: TURN
Distance for robot to turn to the right

\[ M_{\text{SOFT\_RIGHT}} = 0.3 \]
Crisp domain: TURN
Distance for robot to turn to the right

\[ M_{\text{SOFT\_RIGHT}} = 0.3 \]

\[ M_{\text{HARD\_RIGHT}} = 0.7 \]
Defuzzification

• Now we have an output that is a fuzzy variable, but we need to convert it to a crisp value to actually send the motor commands.

• Several alternatives:
  – Take Centroid (along Crisp axis) of Blended Area
  – Take Centroid of Largest Area
  – Weighted Means in area of overlap
Defuzzification: Blended Centroid

Crisp domain: TURN
Distance for robot to turn to the right

\[ C = \frac{\int M_A(X)X}{\int M_A(X)} \]

\[ C \approx \frac{174.6}{3.2} = 54.6 \]
There can be some problems…

Consider the output of an avoid…

And where would the centroid be?

Consider Centroid of Largest Area (CLA)
Defuzzification: Largest Centroid

Crisp domain: TURN
Distance for robot to turn to the right

\[ C = \frac{\int M_A(X)X}{\int M_A(X)} \]

\[ C \approx 61.7 \]
Defuzzification: Weighted Means

Crisp domain: TURN
Distance for robot to turn to the right

\[ C = 0.3(18) + 0.7(72) = 55.8 \]
Back to Swerve

• And so we get an answer!

Dist=3m

PS

MS

PROXIMITY

SOFT_RIGHT=0.3

HARD_RIGHT=0.7

\[ V_{\text{dir}} = 54.6 \]

\[ V_{\text{magr}} = 1 \]
• And so we get an answer!

\[
\begin{align*}
\text{PROXIMITY:} & \quad \text{dist}=3\text{m} \\
\text{PS} \quad \text{MS} & \\
\text{SOFT\_RIGHT}=0.3 \\
\text{HARD\_RIGHT}=0.7 \\
\end{align*}
\]

\[
\begin{align*}
V_{\text{dir}} &= 54.6 \\
V_{\text{magr}} &= 1
\end{align*}
\]
Conjunction, Disjunction, Negation and Hedges

If PROXIMITY is CLOSE AND OPEN is RIGHTSIDE
   TURN is HARD_RIGHT

If PROXIMITY is CLOSE AND OPEN is LEFTSIDE
   TURN is HARD_LEFT

If PROXIMITY is FAR AND OPEN is RIGHTSIDE
   TURN is SOFT_RIGHT

If PROXIMITY is FAR AND OPEN is LEFTSIDE
   TURN is SOFT_LEFT

If PROXIMITY is CLOSE AND NOT (OPEN is RIGHTSIDE OR OPEN is LEFTSIDE)
   TURN is VERY HARD_RIGHT
Evaluation

• **A AND B is minimum(A, B)**
  – PROXIMITY is CLOSE= 0.7
  – OPEN IS RIGHTSIDE= 0.3
  – If PROXIMITY is CLOSE AND OPEN is RIGHTSIDE= 0.3

• **A OR B is maximum(A, B)**
  – PROXIMITY is CLOSE= 0.7
  – OPEN IS RIGHTSIDE= 0.3
  – If PROXIMITY is CLOSE OR OPEN is RIGHTSIDE= 0.7

• **NOT A is 1-A**
  – OPEN is RIGHTSIDE=0.2
  – NOT OPEN is RIGHTSIDE=1-0.2=0.8
  
  (1.0 is full membership for our examples)
Example

If PROXIMITY is CLOSE AND NOT (OPEN is RIGHTSIDE OR OPEN is LEFTSIDE)

Where

OPEN is RIGHTSIDE=0.1
OPEN is LEFTSIDE=0.15
PROXIMITY is CLOSE=0.95
If PROXIMITY is CLOSE AND NOT (OPEN is RIGHTSIDE OR OPEN is LEFTSIDE)

TURN is VERY HARD_RIGHT

OPEN is RIGHTSIDE=0.1
OPEN is LEFTSIDE=0.15
PROXIMITY is CLOSE=0.95

Moves the fuzzy set over
Summary

• Many packages have fuzzy logic: Java, MATLAB

• Fuzzy works by
  – Fuzzification of crisp values
  – Application of rules, each of which has a strength
  – Defuzzification output of rules to produce a crisp output

• Problems include
  – Results may not be what was expected
  – The number and shape of sets impact behavior